Intermediate Mathematical Challenge

Solutions 2025

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For reasons of space, these solutions are necessarily brief.

There are more in-depth, extended solutions available on the UKMT website, which include some exercises for further investigation:
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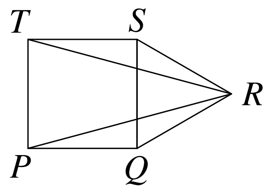
1. A Note that $2 \times 5 - 8 \times 3 = 10 - 24 = -14$; $3 \times 4 - 7 \times 4 = 12 - 28 = -16$; $4 \times 3 - 6 \times 5 = 12 - 30 = -18$; $5 \times 2 - 5 \times 6 = 10 - 30 = -20$; $6 \times 1 - 4 \times 7 = 6 - 28 = -22$.
Therefore expression A has a value which is closest to zero.

2. D Note that 26, 52 and 13 are all multiples of 13. So 2 600 000, 52 000 and 130 are also all multiples of 13. Therefore 2 652 134 leaves a remainder of 4 when divided by 13.

3. E As $PQST$ is a square, $SQ = TS$. Also, as triangle QRS is equilateral, $SQ = SR$. Therefore STR is an isosceles triangle in which $TS = SR$. So $\angle TRS = \angle STR = ((180 - 90 - 60) \div 2)^\circ = 15^\circ$. Similarly, $\angle PRQ = 15^\circ$. Hence $\angle PRT = (60 - 2 \times 15)^\circ = 30^\circ$.



4. D Let the number of cats be n. Then the number of tails is n, while the number of legs is $4n$.
So $4n = n + 12$. Hence $3n = 12$, that is $n = 4$.
Therefore the total number of ears which the cats have is $2n = 8$.

5. E $5 \div (((5 \div 5) \div (5 \div 5)) \div 5) = 5 \div ((1 \div 1) \div 5) = 5 \div (1 \div 5) = 5 \div \frac{1}{5} = 5 \times 5 = 25$.

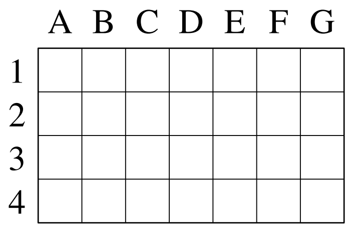
6. C Given that $3n + 7$ is even, we may deduce that $3n$ is odd and therefore that n is odd. For odd n, an expression of the form $pn + q$ is odd if p is even and q is odd, or if p is odd and q is even. Of the options given, only $3n + 2$ satisfies this requirement.

7. B Clearly, exactly one of the dolphin and the shark is telling the truth, though we are unable to say which it is. However, this means that the octopus is not telling the truth, which in turn means that the starfish is not telling the truth either. So exactly one of the four is telling the truth.

8. A $34\frac{1}{7} \div 17\frac{1}{14} = \frac{239}{7} \div \frac{239}{14} = \frac{239}{7} \times \frac{14}{239} = \frac{14}{7} = 2$.

(*Alternatively, note that* 34 *is twice* 17 *and* $\frac{1}{7}$ *is twice* $\frac{1}{14}$, *so* $34\frac{1}{7} \div 17\frac{1}{14} = 2$.)

9. A Lottie can choose the two rows to be coloured in three different ways: 1 and 3, 1 and 4, 2 and 4. If she chooses to colour column A then the second column may be any of the five from C to G inclusive. Similarly, if she chooses to colour column B then the second column may be any of the four from D to G inclusive. If column C is coloured then Lottie may colour any of the three columns from E to G (note that we have already counted the pair of columns A and C). If column D is coloured then the only possible pairs which have not already been counted are D and F and D and G. Finally, if Lottie colours column E then the only possible new pair of columns is E and G. Therefore, the number of different ways in which Lottie can colour two columns is $5 + 4 + 3 + 2 + 1 = 15$.



So the number of ways in which Lottie can colour two rows and two columns, as required, is $3 \times 15 = 45$.

10. C $\sqrt{2025^2 - 2024 - 2025} = \sqrt{2025(2025 - 1) - 2024} = \sqrt{2025 \times 2024 - 2024} = \sqrt{2024(2025 - 1)}$ $= \sqrt{2024 \times 2024} = 2024$.

11. A Let Emily's two integers be m and n. Then $mn = 360$; $(m + 1)(n + 1) = 400$.
Therefore $mn + m + n + 1 = 400$. So $360 + m + n + 1 = 400$. Hence $m + n = 400 - 360 - 1 = 39$.
So the sum of Emily's two integers is 39.
(*It is left as an exercise for the reader to solve the original two simultaneous equations to find the values of m and n and hence confirm that* $m + n = 39$.)

12. E Let the number of dogs on Jane's farm be n. Then at the start of the week there were $\frac{3n}{5}$ cats. After 32 cats but no dogs arrived, the ratio became 5 : 3, so $\frac{3n}{5} + 32 = \frac{5n}{3}$.

Multiplying throughout by 15: $9n + 480 = 25n$. Therefore $16n = 480$, that is $n = 30$.
So there are 30 dogs on Jane's farm.

13. E The product of the first three numbers is equal to the product of the last three numbers, so $x \times 120 \times 496 = 496 \times 360 \times 48$. Hence, $x \times 120 = 360 \times 48$.

So $x = \frac{360 \times 48}{120} = 3 \times 48 = 144$.

14. A Let x be the recurring decimal $0.\dot{n}$, where n is a single digit. Then $x = 0.nnnnn\ldots$.
Hence $10x = n.nnnnn\ldots = n + x$. So $9x = n$. Therefore $x = \frac{n}{9}$.

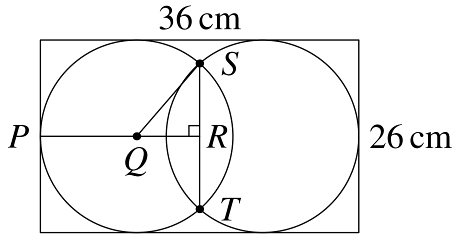
Hence $0.\dot{1} + 0.\dot{2} + 0.\dot{3} + 0.\dot{4} = \frac{1}{9} + \frac{2}{9} + \frac{3}{9} + \frac{4}{9} = \frac{10}{9} = 1\frac{1}{9} = 1.\dot{1}$.

15. D $\frac{3^6 - 3^4}{2^9 - 2^3} = \frac{3^4(3^2 - 1)}{2^3(2^6 - 1)} = \frac{81 \times 8}{8 \times 63} = \frac{81}{63} = \frac{9}{7}$.

16. B Let the number of £1.99 books which David buys be x and the number of £0.99 books he buys be y. Every time David buys a book, he pays £0.01 less than a whole number of pounds and his total cost is £0.44 less than a whole number of pounds. Therefore the number of books which David buys is 44 or 144 or 244 etc. However, as the total cost is less than $144 \times$ £0.99, we can deduce that David buys 44 books. Hence $x + y = 44$. The total cost of x books costing £1.99 and y books costing £0.99 is £$(1.99x + 0.99y)$. So $1.99x + 0.99y = 56.56$.
Therefore $2x + y = 1.99x + 0.99y + 0.01x + 0.01y = 56.56 + 0.01(x + y) = 56.56 + 0.44 = 57$.
So $2x + y - (x + y) = 57 - 44$. Therefore $x = 13$. Hence David buys 13 books at £1.99 each.

17. D In Pablo's mixture, the percentage of blue paint which originates from dark green paint is 60% of 60%, that is 36%. Also, 40% of the mixture is made up of light green paint, of which 40% is blue. So the percentage of blue paint in the mixture which originates from light green paint is 40% of 40%, that is 16%. So the percentage of blue paint in Pablo's paint is (36+16)%, that is 52%, and the remaining 48% consists of yellow paint.
Therefore the required ratio is 52 : 48, that is 13 : 12.

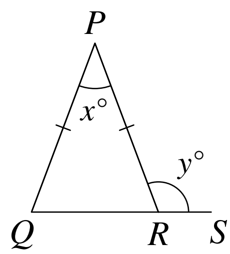
18. E Let the length of ST be $2h$ cm. Note that the radius of each circle is $(26 \div 2)$ cm = 13 cm. So $PQ = QS = 13$ cm. From the symmetry of the diagram we can deduce that PR is half the length of the rectangle, that is 18 cm. Therefore $QR = PR - PQ = (18 - 13)$ cm = 5 cm.



By Pythagoras' Theorem, $QS^2 = QR^2 + SR^2$. Hence $13^2 = 5^2 + h^2$.
So $h^2 = 169 - 25 = 144$. Therefore $h = 12$.
So the distance between the two points where the circles intersect is 2×12 cm, that is 24 cm.

19. D The person who painted twice as many posts in the morning as in the afternoon painted a total number of posts which is a multiple of 3. So he could have been any of the three men. The person who painted three times as many posts in the morning as in the afternoon painted a total number of posts which is a multiple of 4. Hence he is Roy and he painted 36 posts in the morning and the remaining 12 in the afternoon. The person who painted four times as many posts in the morning as in the afternoon painted a total number of posts which is a multiple of 5. Therefore he is Rob and he painted 36 posts in the morning and the remaining 9 in the afternoon. That leaves Rog, who painted 34 posts in the morning and the remaining 17 in the afternoon. So Rob and Roy painted the most fence posts in the morning, since they each painted 36 and Rog painted only 34.

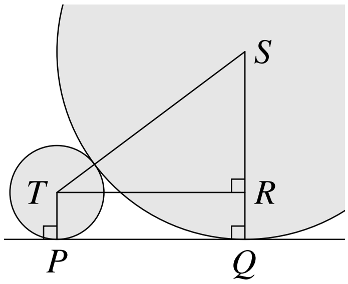
20. B As $PQ = PR$, $\angle PRQ = \angle PQR = ((180 - x)/2))^\circ = (90 - x/2)^\circ$.
Hence $y = 180 - (90 - x/2) = 90 + x/2$ and so, since y is an integer, x must be even. Also, we see that $\frac{y}{x} = \frac{90 + x/2}{x} = \frac{90}{x} + \frac{1}{2}$ and so, for $\frac{y}{x}$ to be as large as possible, x must be as small as possible.
If $x = 2$ then $\frac{y}{x} = \frac{90}{2} + \frac{1}{2}$, which is not an integer.
However, if $x = 4$ then $\frac{y}{x} = \frac{90}{4} + \frac{1}{2} = \frac{45}{2} + \frac{1}{2} = \frac{46}{2} = 23$.



So the largest possible value of $\frac{y}{x}$ is 23. It occurs when $x = 4$ and $y = 92$.

21. B Note that $8^8 = (8^2)^4 = 64^4$. Now the units digit of $(64)^4$ is the same as the units digit of $4^4 = 16^2$. Hence the units digit of 8^8 is the units digit of 6^2, namely 6.
Therefore 8^8 may be written in the form $10k + 6$, where k is an integer.
So $8^8 = 5(2k + 1) + 1$ and therefore leaves remainder 1 when divided by 5.

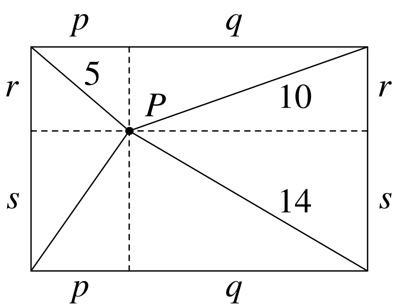
22. **D** Let the centres of the spheres be T and S and let the points where the spheres touch the table be P and Q, as shown.
Let R be the foot of the perpendicular from T to SQ.
So the required distance is PQ, which is equal to TR.
Let this distance be d cm.
Now $TS = (4 + 16)$ cm $= 20$ cm; $SR = SQ - RQ = SQ - TP$
$= (16 - 4)$ cm $= 12$ cm.
By Pythagoras' Theorem, $TS^2 = TR^2 + SR^2$. So $20^2 = d^2 + 12^2$.
Hence $d^2 = 20^2 - 12^2 = 400 - 144 = 256$. So $d = \sqrt{256} = 16$.



Therefore the distance between the points where the spheres touch the table is 16 cm.

(*Note that* $20 = 4 \times 5$ *and* $12 = 4 \times 3$, *so RST is a* 3, 4, 5 *triangle. Hence* $d = 4 \times 4 = 16$.)

23. **D** Let p, q, r, s be the lengths, in cm, of the line segments shown in the diagram. Then the distance from P to the fourth corner is $\sqrt{p^2 + s^2}$.
By Pythagoras' Theorem, $p^2 + r^2 = 5^2 = 25$;
$q^2 + r^2 = 10^2 = 100$ and $q^2 + s^2 = 14^2 = 196$.
Therefore $p^2 + r^2 + q^2 + s^2 - (q^2 + r^2) = 25 + 196 - 100 = 121$.
So $p^2 + s^2 = 121$.

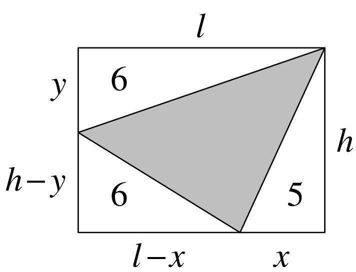


Hence the distance, in cm, from P to the fourth corner is $\sqrt{121} = 11$.

24. **C** Let the cost, in pounds, of a white rose, a yellow rose and a red rose be w, y, r respectively.
Then $2w + y = 5 \; \ldots [1]$; $2w + 3r = 10.5 \; \ldots [2]$ and $3y + 2r = 11 \; \ldots [3]$.
$[2] - [1]$ gives $3r - y = 5.5 \; \ldots [4]$; $[4] \times 3$ gives $9r - 3y = 16.5 \; \ldots [5]$;
$[3] + [5]$ gives $11r = 27.5$. So $r = 2.5$.
$[1] + [2] + [3]$ gives $4w + 4y + 5r = 26.5$. Therefore $4(w + y + r) + 2.5 = 26.5$.
So $w + y + r = (26.5 - 2.5) \div 4 = 6$.
Hence the total cost of one red rose, one white rose and one yellow rose is £6.

(*As an exercise, it is left to the reader to calculate the values of w and y and hence to confirm that the total cost of one rose of each colour is indeed £6.*)

25. **B** Let the area of the rectangle be A cm^2, its length be l cm and its height be h cm. Then $A = lh$.
Also, let x and y be the distances, in cm, shown in the diagram.
The formula for the area of a triangle gives the equations:
$\frac{1}{2}ly = 6$; $\frac{1}{2}hx = 5$ and $\frac{1}{2}(l - x)(h - y) = 6$.
Therefore $ly = 12$; $hx = 10$ and $(l - x)(h - y) = 12$.



Hence $lh - ly - hx + xy = 12$, that is $lh - ly - hx + \dfrac{10}{h} \times \dfrac{12}{l} = 12$.

So $A - 12 - 10 + \dfrac{120}{A} = 12$.

Rearranging and multiplying throughout by A: $A^2 - 34A + 120 = 0$. Therefore $(A - 30)(A - 4) = 0$.
So $A = 30$ or $A = 4$.
However, $A > 6 + 6 + 5$, so $A \neq 4$.
Hence the area of the rectangle is 30 cm^2.
Therefore the area of the shaded triangle is $(30 - 6 - 6 - 5)$ cm^2, that is 13 cm^2.